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RESEARCH

Modelling of Nonlinear Dynamic of Mechanic Systems with the Force Tribological Interaction

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ABSTRACT

This paper considers the mechanisms with different structure: tribometric device and a mechanism for handling of optical glasses. In the first device, the movement of the upper platform is due to a reciprocating friction interaction. In the second device, the processing of the optical element or group of elements occurs due to the rotational motion. Modelling of the dynamic of these systems with Matlab/Simmechanic allowed carrying out the analysis of dynamic of mechanisms, considering nonlinearity tribological interactions for these systems. The article shows that using of the computer models can effectively carry out the selection of the control parameters to create the desired mode of operation, as well as to investigate the behaviour of systems with nonlinear parameters and processes of self-oscillations. The organization of the managed self-oscillation process is realized to create the relevant high-performance manufacturing, for example, for the processing of optical glasses.

1. INTRODUCTION

In present time, one of the most important problems is the research of the frictional systems, which are based on the interaction of surfaces of different configurations with rough plane or a surface of complex shape. The results of such research can positively influence the development of braking or grinding tools and machinery in general. Feature of such systems is the presence of intermediate liquid and viscous lubricants and emulsions, complicating their theoretical research and prognostics results. Today, the optics manufacture and other high-tech industries use different types of machines: grinding, polishing, lapping, etc. All these machines are united by the same principle of the working bodies, based on the reciprocal shaping and torque transferring from the processed tools to the body by using the frictional forces. This fact impose some difficulties for machine researching and modernization, that include: the development of non-contact methods of control of the surface state, the identification of self-oscillatory processes and their impact on the establishment of processing modes, accounting elastic and thermoelastic deformations, analysis of tool path, construction of feedback algorithms, creating of mathematical models for the dynamic processes prediction, and creating of mathematical models for the magnitude of material removal prediction in the treatment zone.

For the identification of friction and self-oscillation, a research was developed imitating the model of tribometric device "Tribal". This device was created at the Department of Mechatronics in ITMO and patented [1]. The kinematic diagram of the device is shown in Fig. 2. Matlab simulation model of the "Tribal" is shown in Fig. 3a. These models are obtained for different operating conditions, and were compared with experimental values.

For studying the dynamic, in grinding and polishing devises for processing optical glass, an actuator imitating model SimMechanics was created, Fig. 4a. The main objective of this study is to investigate the impact on the process fluctuations in friction, to define the dynamics of machine for grinding and polishing operations, by free lapping and studying level changes of surface roughness using a non-linear friction law.

2. THEORETICAL JUSTIFICATION

Dynamic interaction of rough contacting surfaces is carried out in the relative motion of solid bodies. The deformation zone of reciprocal contact occurs in conjunction with the molecular interaction processes and the process of heat deterioration. According to the theory of Bowden and others [2], friction force is mainly due to two reasons: on the one hand, there is the cutting of adhesion links formed in the areas of actual contact of solid bodies, and on the other hand, there is the ploughing of less rigid surface material of the interacting bodies [3,4]. Mediums, located between the contacting surfaces, have a decisive influence on the friction between the rough surfaces and the physical-chemical processes in the environment. For the automatic control and predicting of the wear processes, a various models of friction are of interest. At present, there are many physical, mathematical, and analogue simulation models, which describe the phenomena occurring in the contact surfaces with different types of friction [5,6]. Friction is interpreted as a physical interface between the contacting surfaces. Static friction models include: models of dry and viscous friction on the basis of Coulomb's law, Karnopp model, and Armstrong model for describing the dynamic processes in the classical model of friction. To describe the process of friction, a various nonlinear models that use power, quadratic resistance, as well as methods of catastrophe theory, are also used.

In the work [1], in the study of dynamical tribological interaction by using the device "Tribal", it was found that the surface damage begins when the dynamic system pass the bifurcation point. During the tests, conducted systematically, assessing dynamical models correspond to the process of friction. At each stage, the system identified two characteristics: the impulse response and the identity of a transitional function. Thus, in sliding friction it was registered the second order phase transition, or Hopf bifurcation, after which begins the process of deterioration. Calculated scale of the phenomenon allows considering the emergence of an additional degree of freedom (Fig. 1). Based on these results, it was decided to use the theory of catastrophes for studying the problem of friction in the processing of optical glasses.

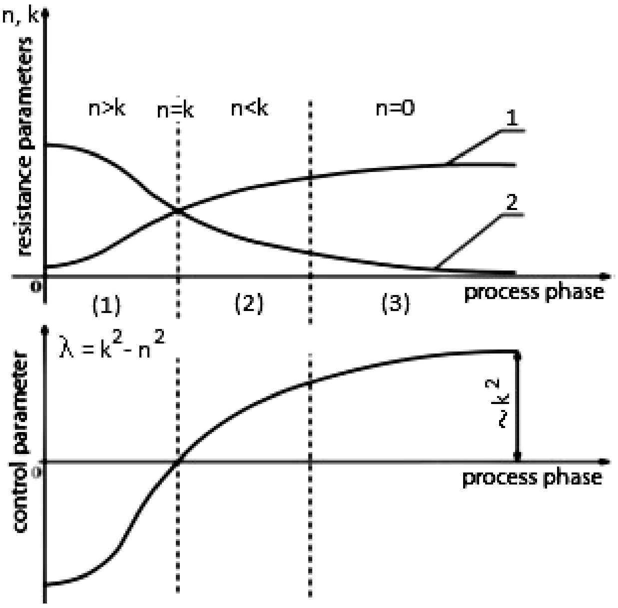


Fig. 1. Changing the damping coefficients and natural frequencies: n – damping factor, k – natural vibration frequency, $\lambda = n^2 - k^2$ – natural parameter.

3. MODELLING THE DYNAMICS OF THE TRIBOMETRIC DEVICE "TRIBAL"

Tribometric device (Fig. 2) consists of a base 1, two platforms: the lower 2, and the upper 3. Platforms are capable of reciprocating movement along the rails 4 and 5. The upper platform can move in the vertical direction due to the guide 7. On the upper platform acts forces of the springs 6, mounted on the rail 5, as well as vertical force (P). Lower platform is driven by the motor 9 via a crank mechanism 8 and transmits movement to the upper platform due to frictional forces. Under the action of springs, the upper platform tends to return to its original position and thus oscillates. Platforms positions are determined by linear encoders.

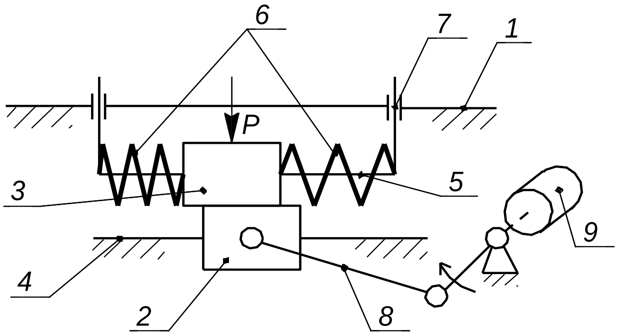


Fig. 2. Kinematic scheme of the tribometric device: 1 – base; 2 – lower platform; 3 – upper platform; 4 – lower platform rail; 5 – upper platform rail; 6 – springs; 7 – vertical guide; 8 – crank mechanism; 9 – engine.

Figure 3a shows the animated SimMechanics model of tribometric devise built by using a blocks Matlab/Simulink/ Simmechanics. Vertical force in the model is taken constant and given by the mass of the upper platform. Spring and damper block simulates the elastic-dissipative forces, depending on the set of coefficients of elasticity and damping. Friction force depends of the speed of the platforms relative motion, and is given by the block Joint Stiction Actuator (drive unit with “sticking”). The plate movements define the group of sensors blocks. More details of the model of friction are considered in [4]. To build the model, the equations for frictional relaxation self-oscillations are used, taken from [4,7]. For the modelling of a system, standard blocks of package Simulink/Simmechanics are used, and considered such factors as the difference in the coefficients of friction for the rest and sliding, the dependence of the friction coefficient on the relative velocity, the coefficients of elasticity and the damping of the elastic system. The results of calculation and simulation are shown in Fig. 3b.

Resulting graphs show the performance of the tribometric system. Under the action of friction, the upper platform, lying on the lower, is involved in a joint motion. The spring, fixed to the upper platform, when the resilient force exceeds the force of static friction, shifts upper platform relatively to the lower. The graphs show that the simulation model data correlate well with the experimental ones. The degree of convergence of the results is approximately 85 %. The results of the friction model in the presence of frictional self-oscillations are identical with the analytical solution given in [8].

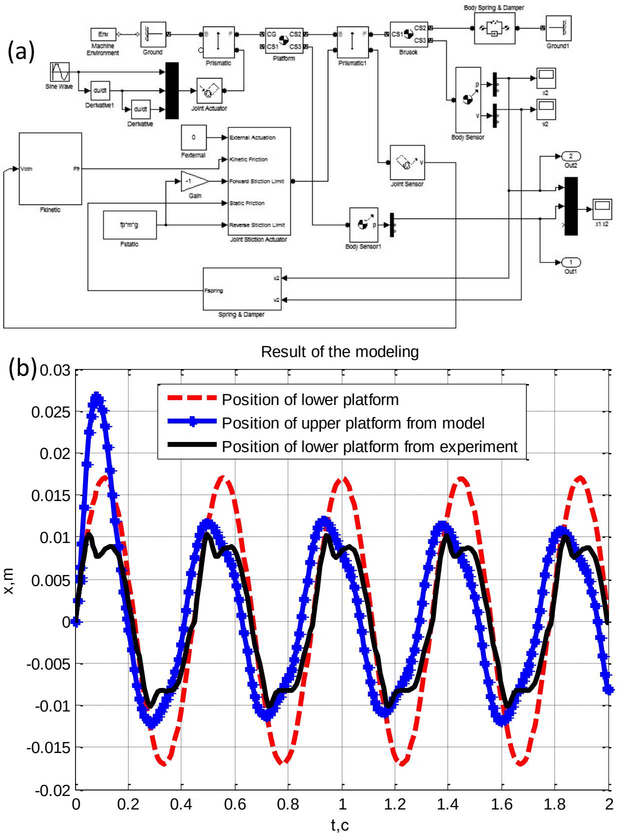


Fig. 3. (a) animated model SimMechanics and (b) comparative graph of experimental data and model data.

The considered model of friction can be used to study the dynamics of various mechanical systems, where it is important to consider the effect of friction. Research of the dynamic processes of such systems allows to make the adjustments to their work and to choose the most effective parameters to improve the efficiency of their use.

4. MODELLING THE WORKING ASSEMBLY OF GRINDING WITH NONLINEAR FRICTION

Current level of development of the optical production requires continuous upgrading of the equipment, introduction of the automation, with the use of computer technology, and research processes of the shaping surfaces. It is necessary to control the following parameters: the deviation of surface shape from the desired; the deviations of mutual disposition of the polishing surfaces relative to baseline, the change of surface roughness, the quality of the constructed optical surfaces image [3,4]. These issues were engaged at different times by the following scientists L.S. Tsesnek, V.A. Smirnov, A. Bardin and others. Exception is machines whose principle is based on forced shaping details. Almost all of the above mentioned problems require a coherent analysis of the dynamics and kinematics in general. The relevance of these problems is mentioned in [3-9]. Using the experience of research described in the Section 2, a simulation model of the actuator for processing of optical elements (Fig. 5) is created.

Figure 4a shows a functional diagram of the mechanism for polishing of the optical glasses. Scheme is conditional and includes: gear and the spindle motor D_1 on one side, and crank mechanism for the leashes swing and motor D_2 on the other side. There are lower faceplate 1 fixed on it the workpiece 2, the torque transmitted from the engine D_1 to lower faceplate. Engine D_2 transmits torque to the joints 5 and 6, which provide the necessary law of motion of the upper faceplate (polishing). It should be noted that, in order to prevent the discontinuity of the kinematic connections between the elements 3 and 5, as well as 2 and 3, the power P is introduced into the scheme. In this case, there is a force connection between the operating elements of the bodies in the kinematic scheme. The entire mechanism is immersed in the emulsion tank with circulating emulsion 4, which is continuously supplied to the contact surface. It cleans them from wear products during polishing.

Dynamic processes occurring in any mechanisms essentially depend on the properties of its mechanical parts. Therefore, it is necessary to analyze the movable joints of the kinematic chain in mechanisms. Thus, the shaft 2 transmits the rotary motion from the rack O_2 (Fig. 4b) and forms a fifth-grade kinematic pair with a conditional rack O_2, forcing specified rotation around axis of the mechanism. The shaft with a crank forms a fifth-grade kinematic pair and allows the machine to rotate around axis. Crank and polishing pad form a third kinematic pair [7]. Third-class couple B allows the machine to perform the rotation around axis. Due to the hinge B, the inner surface of the tool is mounted itself on the workpiece surface. Polishing pad and the workpiece forms a third-class kinematic pair A, and rotation around axes. The workpiece is rigidly fixed to the lower face plate, which is immovably fixed to the shaft 1. The shaft transmits rotary motion of the rack O_1, and forms one fifth-grade kinematic pair with a conditional rack, reporting to the mechanism a specified rotation around the axe. The constant contact, by the force bridging, is provided in the kinematic pairs A and B.

Characteristic movement of the engine D_2 is considered as reciprocating-rotational around the axis. Considering the kinematic scheme (Fig. 4b), we assume that the input links are shafts 1 and 2, and the output link are polishing tools. The mechanism has three degrees of freedom. Consequently, to determine the movement of all links, three generalized coordinates should be given. Make sure to consider that the mechanism for the polishing of optical glasses relates to mechanisms with variable mass units. This is due to the mutual wear of a workpiece and a pad. During the research, the mechanism is necessary to produce wear. Mechanical part is conveniently considered as a system with holonomic, stationary and ideal retaining constraints. This is also applied to the system of the generalized driving forces, acting on the links of the mechanism and the resistance forces, by the links of the actuators [7,10].

Suppose that all the kinematic pairs of the mechanism, except pairs A and B, are ideal, and all links rigid, then:

$$L_S\{q_1,...,q_n,Q_1,...,Q_l\}. \tag{1}$$

In this case, to describe the dynamics of the mechanism it is sufficient to introduce the three generalized coordinates:

$$\begin{cases} \varphi_{z_{01}} = \text{const} \\ \varphi_{z_{02}} = \text{const}; \\ \delta_{y_3} = f(t) \end{cases} \begin{cases} \varphi_{x_1} = \varphi_{x_2} = q_1 \\ \varphi_{y_1} = \varphi_{y_2} = q_2\,, \\ \varphi_{z_2} = q_3 \end{cases} \tag{2}$$

where: z_{01}, $\varphi_{z_{01}}$ and z_{02}, $\varphi_{z_{02}}$ are constant, as engine performance is considered to be ideal.

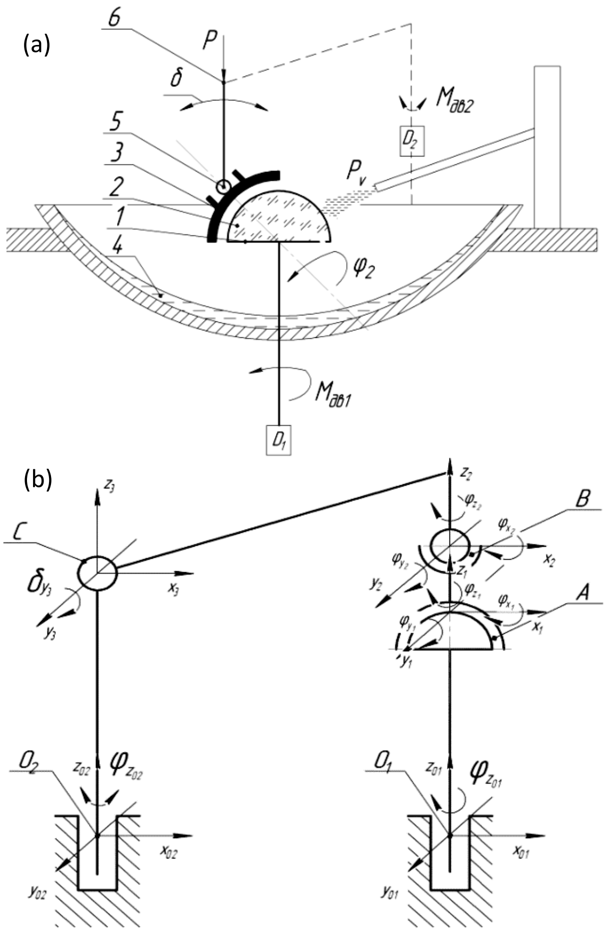


Fig. 4. (a) functional diagram of an optical glass polishing assembly and (b) kinematic diagram of a polishing optical glasses assembly.

Figure 5 shows the animated Matlab/Simulink/ SimMechanics model of working mechanism for processing of optical elements by free lapping. A nonlinear friction law, proposed in [1], is used in the simulation. It is believed that c and b parameters are related to the characteristics of the intermediate layer. Their influence affects the performance of the working mechanism for the optical element polishing.

The comparison of the tools phase trajectory was made by using the data, obtained by the theoretical methods and from the model. The comparison showed that the system with linear friction law does not coincide with the analytical findings. With the gradual introduction of nonlinear coefficients, since the Kelvin-Voigt model, the results are improved. In the process of modelling of the dynamics, system behaviour changed from steady-state (where the phase space subsystems constituted the center) to unstable (where the phase space subsystems has adapted to the steady focus, and then in the unstable focus), and system acquired a damped oscillatory.

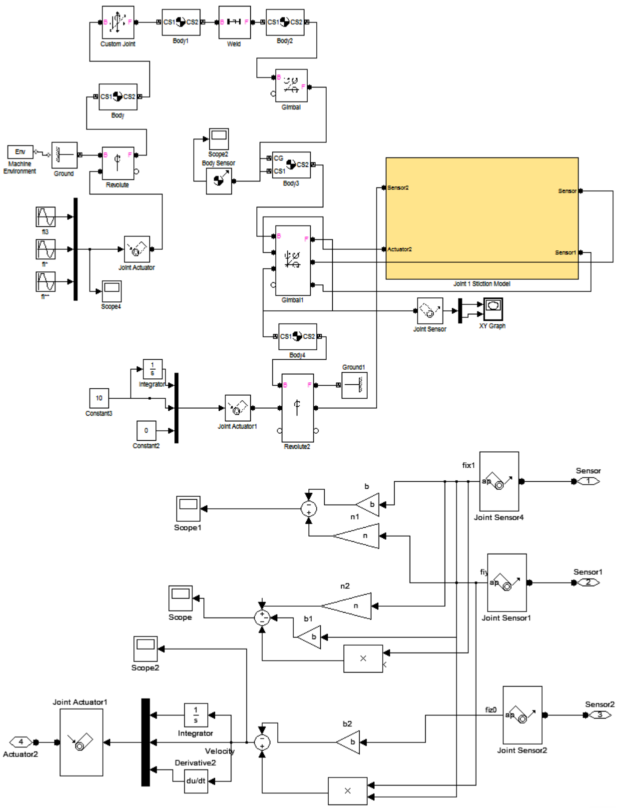


Fig. 5. The SimMechanics animated model of the mechanism for the optical element polishing.

As the extent, when the torque M is applied to the lower faceplate, the system becomes unstable in all cases. This phenomenon takes place when there are equality of magnitude of damping and stiffness component. It follows that the use of extremely viscous fluids for polishing leads to the irrational use of the system driving forces. Form of all phase trajectories of the third body (Fig. 6) indicates the presence of self-oscillating process.

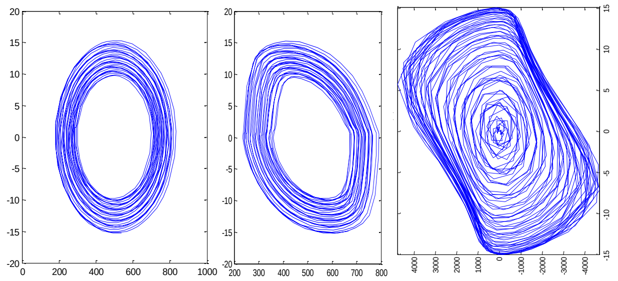


Fig. 6. The results of the SimMechanics animated model of the mechanism for the optical element polishing.

The model actuator spindle block of the grinding machine in processing of flat optical elements was synthesized next (Fig. 7).

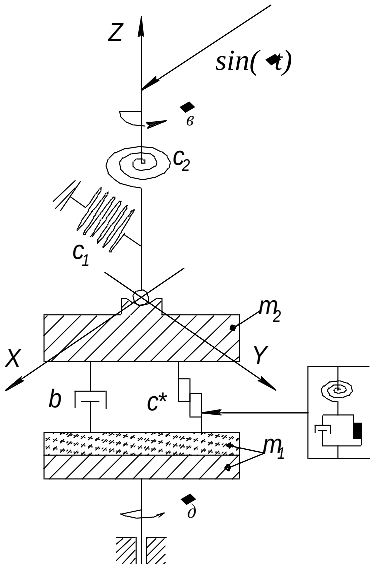


Fig. 7. The equivalent circuit of the actuator for flat optical elements

The lower chuck, of mass m_1, aligned with the optical element was fixed on it. At the bottom faceplate acts an input torque from the motor M_1. The angular velocity of the upper faceplate of mass m_2 is transmitted through the elastic-dissipative layer. It is characterized by the two components: the dissipative body, which has a viscosity coefficient b, and the visco-plastic body *c**, whose properties are expressed by equation (3):

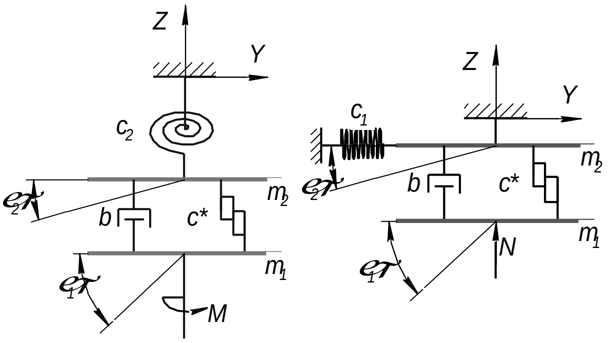
$$q_n + m \cdot \dot{q}_n = c^* \qquad (3)$$

where: μ is the plastic viscosity and q_n is the generalized coordinate.

The spindle elasticity, pronounced by the stiffness coefficient of the elastic element c_2, allows the system to work on twisting. The stiffness coefficient of the elastic element c_1 provides to the system the ability to move under the influence of $P(t)$ of the motor M_2. This effect is permanent with the alternating sign, and can be expressed by the equation (4):

$$P(t)_{\text{pov}} = A \cdot \sin(\omega t)\,. \qquad (4)$$

The equations of motion for a pair tool-workpiece can be found by expanding the system into two equivalent schemes (Fig. 8). This corresponds to the independent degrees of freedom (DOF): rotation about the *Z* axis and displacement along the *XY* plane. The directions of these DOF are shown in Fig. 7. Elastic-dissipative layer, which is located between the bodies, work in all directions of motion of tool-workpiece pair: the twisting and moving, so we can assume that the intermediate layer is isotropic. Equivalent scheme (Fig. 8) is described by a system of two equations with three additional (equation 5):



(a) (b)

Fig. 8. The equivalent schemes for the calculation: (a) scheme of the executive system axis on the *Z*-axis and (b) scheme of the executive system axis on the *Y*-axis

$$\begin{cases} J_2\ddot{\varphi}_2 + (\varphi_1 + \mu \cdot \dot{\varphi}_2)(\dot{\varphi}_2 - \dot{\varphi}_1) + c_2\varphi_2 + \\ \qquad + b(\dot{\varphi}_2 - \dot{\varphi}_1) = M_{\text{dv}}(t) \\ m_2\ddot{y}_2 + (y_1 + \mu \cdot \dot{y}_2)(\dot{y}_2 - \dot{y}_1) + c_1 y_2 + \\ \qquad + b(\dot{y}_2 - \dot{y}_1) = P(t)_{\text{dv}} \end{cases}, \qquad (5)$$

where: φ_1 is the initial angle of the bottom faceplate (its position is known, i.e. $\varphi_1 = 0$ for $t = 0$).

$$\begin{cases} (\varphi_1 + \mu \cdot \dot{\varphi}_2)(\dot{\varphi}_2 - \dot{\varphi}_1) + b(\dot{\varphi}_2 - \dot{\varphi}_1) = N \\ (y_1 + \mu \cdot \dot{y}_2)(\dot{y}_2 - \dot{y}_1) + b(\dot{y}_2 - \dot{y}_1) = F_{\text{tr}} \end{cases}. \qquad (6)$$

Thus, in the systems of equations (5) and (6) there are five unknowns: φ_2, y_2, y_1, F_{tr} and *N*. It should be noted that the absence of contact of the upper faceplate with workpiece, by elastically-dissipative layer, leads to the absence of mutual abrasion of the surfaces. The contact surface is the main kinematic condition. In case of breakage of the contact, the inertial motion of the emulsion layer should be also neglected due to its small thickness and speed. Coordinates *X* and *Y* are interrelated in the equation (7):

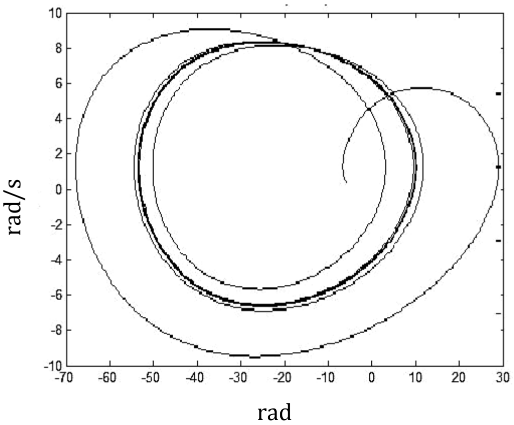
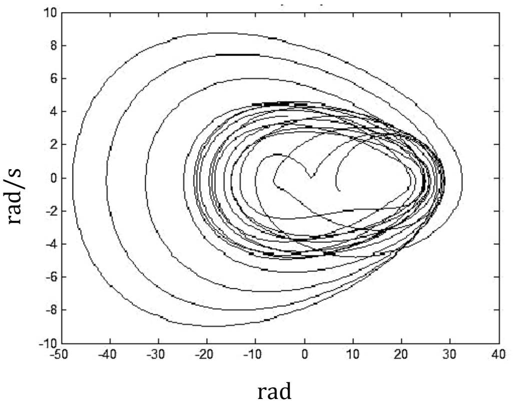
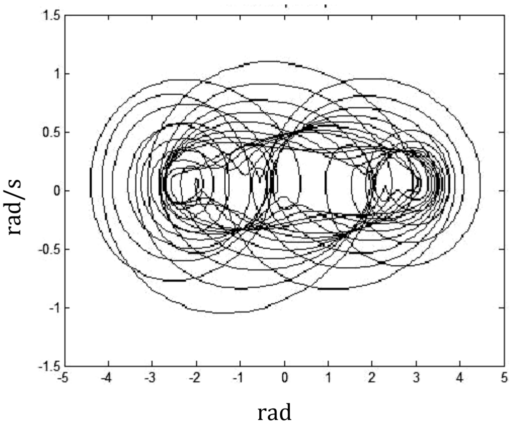
$$x^2 + y^2 = L^2\,. \qquad (7)$$

where: *L* is the length of the carrier.

Elastic-dissipative characteristics of the system are calculated from the theoretical calculations. In addition, it is assumed that the evolution of the system parameters occurs considering the three main stages, presented in Fig. 6.

Initial parameters are selected on the basis of a flat lens processing mode, considered in [1]. The modelling results obtained by the equation are presented in Table 1.

Table 1. The modelling results.

c, b	
c < b	
c = b	
c > b	

5. CONCLUSION

On the basis of experimental studies by using the tribometric device “Tribal”, a SimMechanics animated simulation model was developed. This allowed comparison of the experimental and theoretical results, as well as the establishment of their degree of convergence. After the positive results, a SimMechanics animated model was created for the main mechanism for processing of the optical elements. The necessity of this approach is due to the fact that the technological features of the machines do not allow us to study the internal processes experimentally. Several models have been created for different configurations of the machine and were compared [10-12]. The theoretical calculations are carried out with picked laws of friction, for various stages of processing. Research of the friction models is done by using of the two systems: the model “Tribal” and the model of machine for the processing of optical elements. In the future, it is planned to make the selection of the control parameters to determine the optimal properties of the polishing suspension in relation to high-speed mode of the machine.

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